Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

3. In example 3, if 5 cars enter the lot on the average, what is the probability that during any given minute 6 or more cars will enter? First guess. Compare with example 3.

This problem has as its subject the same one that example 3 has. It is dealing with the Poisson distribution.

```
Probability[x ≥ 6, x ≈ PoissonDistribution[5.]]
```

```
0.384039
```

5. Five fair coins are tossed simultaneously. Find the probability function of the random variable X = Number of heads and compute the probabilities of obtaining no heads, precisely 1 head, at least 1 head, not more than 4 heads.

I should note that the “simultaneously” in the problem description has no effect. The same results are expected if the flips take place one per hour as well as all five simultaneously.

```
heads[n_] := BinomialDistribution[n, 0.5]
```

```
Probability[x == 0, x ≈ heads[5]]
```

```
0.03125
```

```
Probability[x == 1, x ≈ heads[5]]
```

```
0.15625
```

```
Probability[x ≥ 1, x ≈ heads[5]]
```

```
0.96875
```

```
Probability[x < 5, x ≈ heads[5]]
```

```
0.96875
```

The values in green above match the text answers.

7. Let X be the number of cars per minute passing a certain point of some road between 8 A.M. and 10 A.M. on a Sunday. Assume that X has a Poisson distribution with mean 5. Find the probability of observing 4 or fewer cars during any given minute.

It took me awhile to see that the two hours or 120 minutes is meaningless, because the problem is simply talking about cars per minute throughout. For strictly less than 4 cars seen, I get the text answer, green below, but taking the problem description literally and allowing four cars, as well as fewer than four, I get the yellow cell's answer.

```
Probability[x < 4, x ≈ PoissonDistribution[5.]]
```

```
0.265026
```

```
Probability[x ≤ 4, x ≈ PoissonDistribution[5.]]
```

```
0.440493
```

9. Rutherford-Geiger experiments. In 1910, E. Rutherford and H. Geiger showed experimentally that the number of alpha particles emitted per second in a radioactive process is a random variable X having a Poisson distribution. If X has mean 0.5, what is the probability of observing two or more particles during any given second?

```
Probability[x ≥ 2, x ≈ PoissonDistribution[0.5]]
```

```
0.090204
```

The green cell above agrees with the answer in the text.

10. Let $p = 2\,\%$ be the probability that a certain type of lightbulb will fail in a 24-hour test. Find the probability that a sign consisting of 15 such bulbs will burn 24 hours with no bulb failures.

11. Guess how much less the probability in problem 10 would be if the sign consisted of 100 bulbs. Then calculate.

```
Clear["Global`*"]
```

```
Probability[x == 0, x ≈ BinomialDistribution[100, 0.02]]
```

```
0.13262
```

The green cell above agrees with the answer in the text.

13. Suppose that a test for extrasensory perception consists of naming (in any order) 3 cards randomly drawn from a deck of 13 cards. Find the probability that by chance alone, the person will correctly name (a) no cards, (b) 1 card, (c) 2 cards, (d) 3 cards.

Hypergeometric distribution is the one used for drawing marbles out of a bag. In that case I might be looking for black or red marbles. The setup below is exactly the one I would use if the bag contained 10 black marbles and 3 red, and I was interested in the probabilities about drawing out red marbles on 3 draws. In the present problem, there are 13 cards in the deck, 3 of which are marked by the invisible distinction of being the ones which will demand identification on being drawn, and 10 of which will have their anonymity preserved by fate. The values of the two number 3s on the d line are locked together, because it is the first 3, the number of draws, that imparts the magical "guessable" character to the second.

```
d = MultivariateHypergeometricDistribution[3, {10, 3}];
Probability[x == 3 && y == 0, {x, y} ≈ d]
```

$$\frac{60}{143}$$

```
N[%]
```

```
0.41958
```

```
Probability[x == 2 && y == 1, {x, y} ≈ d]
```

$$\frac{135}{286}$$

```
N[%]
```

```
0.472028
```

```
Probability[x == 1 && y == 2, {x, y} ≈ d]
```

$$\frac{15}{143}$$

```
N[%]
```

```
0.104895
```

```
Probability[x == 0 && y == 3, {x, y} ≈ d]
```

$$\frac{1}{286}$$

```
N[%]
```

```
0.0034965
```

The green cells above match the answer in the text.

15. Suppose that in the production of 60-ohm radio resistors, nondefective items are those that have a resistance between 58 and 62 ohms and the probability of a resistor's being defective is 0.1%. The resistors are sold in lots of 200, with the guarantee that all resistors are nondefective. What is the probability that a given lot will violate this guarantee? (Use the Poisson distribution.)

The problem gives more information than is needed. The only things that are needed are the probability of a defect on a resistor, and the lot size. The following cells were modeled after a Mathematica docs example on the Poisson distribution.

I need an expression of the probability of defects.

```
Mean[PoissonDistribution[λ]] == 1/1000
```

$\lambda == \frac{1}{1000}$

```
NSolve[%, λ]
```

{{λ → 0.001}}

I make up my own distribution based on Poisson.

```
resisterDefectDistribution[p_] :=
 Evaluate[PoissonDistribution[λ p] /. First[%]]
```

I test it out to see that the chance of a non-defective single resistor returns the mean I quoted.

```
PDF[resisterDefectDistribution[1], 0]
```

0.999

I query the chance of a perfect lot.

```
PDF[resisterDefectDistribution[200], 0]
```

0.818731

I rearrange to express the chance of a lot with at least one defect.

```
1 - %
```

0.181269

The green cell above matches the answer in the text.